Exercise 14

Graph the two basic solutions along with several other solutions of the differential equation. What features do the solutions have in common?

$$4\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 0$$

Solution

This is a linear homogeneous ODE with constant coefficients, so its solutions are of the form $y = e^{rx}$.

$$y = e^{rx} \quad \to \quad \frac{dy}{dx} = re^{rx} \quad \to \quad \frac{d^2y}{dx^2} = r^2e^{rx}$$

Plug these formulas into the ODE.

$$4(r^2e^{rx}) - 4(re^{rx}) + e^{rx} = 0$$

Divide both sides by e^{rx}.

$$4r^2 - 4r + 1 = 0$$

Solve for r.

$$(2r-1)^2 = 0$$

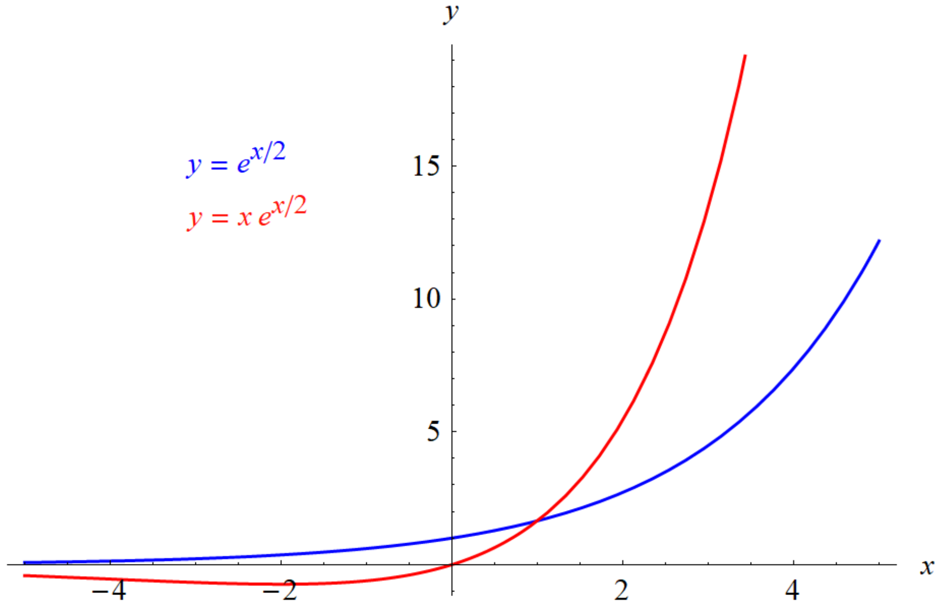
$$r = \left\{\frac{1}{2}\right\}$$

Two solutions to the ODE are $e^{x/2}$ and $xe^{x/2}$. By the principle of superposition, then,

$$y(x) = C_1e^{x/2} + C_2xe^{x/2},$$

where C_1 and C_2 are arbitrary constants.

Below is a graph of these two solutions.



Both solutions blow up as $x \to \infty$.